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ABSTRACT

A technique for the analysis of ecosystems developed by Odum focuses on the nature of interactions which take place within the system being considered. This technique can be used to assess the contribution of any variable to an interaction. Using data derived from previous research conducted by Feather, the authors employ this technique to examine the way in which probability estimates and actual achievement interact as a system. Specifically, the data were studied to determine how achievement and probability of success varied over time for each treatment group. The interactions were incorporated into a model allowing symbolic representation to be translated into a useable mathematical form. Thus each dynamic variable in the system can be represented by a differential equation which, when combined, provide data on the changes of each variable over time. (AJL)

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AN ANALYSIS OF INTERACTION OF ANXIETY, ASPIRATION LEVEL AND ABILITY DERIVED FROM ECOLOGICAL SYSTEMS THEORY

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Introduction

A technique for the analysis of ecosystems developed by Odum (1971) has met with considerable success as an empirical predictor of system behavior. The focus of the technique is on the nature of the interactions which take place within the system being considered. Variables within the system are dynamic over time, while influences from outside the system are either fixed or vary in a pre-determined fashion and are not affected by the behavior of the system being considered. The nature of the interactions within the system are governed by the laws of Thermodynamics (conservation and entropy), thus providing guidance from well understood physical principles as well as providing a transition to mathematical representation of the interactions. The use of these principles is currently finding application in information theory (Gatlin, 1972) and economic theory (Georgescu-Roegen, 1975).

In this procedure, a system is defined as a collection of dynamic variables and the interactions between these variables which produce changes over time. Variables outside this framework, which influence the system but which remain unchanged by system behavior, are termed 'forcing functions'. Thus, the system is defined to include those variables within the system as dynamic, if attention is to be focused on these variables and their interactions and not on possible changes which may occur in forcing functions which interact with the dynamic variables but whose behavior is pre-determined. Variables within the defined system which remain constant over time are not specified. In calculations involving thermodynamic principles, variables which are constant are not included in the calculations. Therefore, these are not included in the definition of the system. Should other variables which were considered as constants actually prove to be dynamic, the

system will not function adequately unless the variables are included.

Interactions which take place among variables and forcing functions may involve variations which result from linear or inverse, additive or multiplicative interactions or by interactions which can be represented by trigonometric functions and/or a combination of all types. Therefore, any system for which information exists as to dynamic variables, forcing functions and their interactions can be defined and investigated. Whether or not the system duplicates known variations is then determined by whether or not the interactions taking place within the system are correctly defined.

Definite principles determine the nature of the interactions and the boundary conditions for such interactions. These are derived from thermodynamics but the most important ones may be restated to be more applicable to social science investigations. A conservation principle operates which defines boundaries for interactions. Stated simply, this principle indicates that the sum of all contributions of all variables to an interaction must be equal to the sum of all the results of the interaction plus any losses which occur during the interaction.

A second principle of entropy further refines the conservation principle. It states that the sum of the products must always be less than the sum of the contributions, that is no interaction is perfect - some loss is always associated with an interaction although this may be very small. This loss is in proportion to the difficulty encountered in achieving the interaction. An example of the application of the first and second principle may clarify these. Consider two stimuli, one visual and one auditory, interacting with one another so that the result is additive. The first principle limits the resulting interaction to the sum of the

two stimuli. The second principle reduces the result by some quantity, making the result of the interaction of the two greater than either stimulus alone, but not equal to the sum of the two.

A third principle is a maximizing principle. It states that interactions are maximum over time if the sum of the products of the interactions is 50 percent of the sum of the contributions to the interactions. Many examples of this exist in psychology. Intermediate levels of motivation are optimum and optimum achievement activity occurs at intermediate levels of probability of success. The interaction of student and learning material may well be maximized at intermediate levels. If the student interacts with simple material, the result is diminished because much of the interaction is hasty and material is ignored. Material which is too difficult results in diminished interaction as well, though for different reasons. Maximum interaction over time occurs with materials of intermediate difficulty. Perhaps this principle explains to some extent, one of the shortcomings of programmed material in which the steps are too small. It must be noted that this, we refer here to maximization over time, not to maximum instant results. Maximization over time results from interactions which are neither too easy nor too difficult to attain.

A final major principle is a depreciation principle. This principle states that all dynamic variables require maintenance, or, conversely, if the variable does not interact over time, it depreciates. An obvious example is retention of information. Unless information is used, it depreciates exponentially over time. Maintenance of information counteracts the depreciation resulting in longer retention. Depreciation is also dependent on the magnitude of the stored quantity. More storage requires more maintenance.

Because these principles are based on known physical laws which can be represented mathematically, the model of a system derived from these principles may also be represented mathematically. The contribution of any variable to an interaction is determined by the magnitude of the variable and the interaction which takes place. In some cases, the variable contributes to the interaction; in others it is influenced by the result of an interaction or several interactions. Each of the contributions the variable makes to an interaction can be represented by a mathematical relationship determined by the nature of the interaction. The same can be done for each contribution made to the variable as a result of an interaction. With this information, each dynamic variable in the system can be represented by a differential equation which is a sum of all contributions to the variable minus the sum of all contributions of that variable to interactions within the system. Simultaneous integration of all these equations over time provide data on the changes of each variable over time, and thus data on the behavior of components of the system.

Several advantages accrue from this technique. If data from the model duplicate known data, integration over longer periods of time provide predictions about system behavior in time frames which go beyond the original data. Constants in the differential equations (called transfer coefficients by Odum) indicate the magnitude of the contribution of each variable to an interaction. The reciprocal of the constant is a time constant. This indicates the time required for one complete interaction in relation to other interactions. Finally, it is possible to investigate a wide variety of conditions within the system by beginning

with different initial conditions for the variables. Thus many hypotheses can be tested to determine those facets of a system which are most crucial to system behavior and might provide information concerning those hypotheses which would yield the most significant results when empirical investigations are to be conducted.

Achievement Theory

Achievement Theory, as propounded by Atkinson, is based on the assumption that subjective judgments about probability of success, or the likelihood of attaining a goal, interacts with perception of task stimuli to produce a specific kind of response. It is this postulated interaction or mediation that permits us to label Atkinson's theory as a cognitive, or at least a quasi-cognitive, one.

Briefly, achievement theory can be summarized as follows:

1. According to Murray (1938), an organism responds to the environment because of "needs" which may be labelled as potentialities to respond. The environment may be supportive or it may act to block action. Since the organism acts in specific and voluntary ways in response to the perceived situation, it follows that he is being active in making choices and the overall way in which he makes such choices can be viewed as a personality characteristic.

2. McClelland (1951) refined the work begun by Murray, emphasizing the role of motivation in behavior. McClelland saw motives as being based on emotions which resulted from association between stimulus situations and affective states. Thus anticipated goal reactions initiate and direct behavior through aroused needs. Since such needs fluctuate in intensity, it follows that optimal levels of aroused needs can be postulated. The organism will then act both to reduce needs above optimal level and to increase incoming stimulation if it drops below a certain level.

> This motivational tendency refers to the positive or negative anticipatory goal reactions aroused in situations that involve competition with a standard of excellence, where performance may be evaluated as a success or a failure. (Weiner, 1972, p. 175)

3. The methods used for arousing and measuring the needs for achievement are well known and need not be reviewed here. The Thematic Apperception

Test, developed by Atkinson, is used to obtain protocols describing the content of thought. Experimental procedures increase and decrease perceived estimates of the probability of success, anxiety and achievement. Post test results are analyzed to determine the effects of interactions among these variables.

4. Atkinson (1957) attempted to specify statistical relationships related to achievement theory and to relate these, in turn, to individual differences in achievement needs and their effect on motivation. In addition to person, environment, and experiential variables, Atkinson also dealt with the conflicts associated with choice behavior (Weiner, 1972).

> Further, Atkinson's theory of achievement motivation is influenced by Miller's conflict model. Achievement-related behavior is conceptualized as a resultant of a conflict situation. It is assumed that the cues associated with competition against a standard of excellence arouse both the hope of success and the fear of failure. The strength of the approach tendency toward the goal (the hope of success) relative to the strength of the avoidance tendency (the fear of failure) determines whether the individual will locomote toward or away from achievement-related tasks. (p. 195)

5. The tendency to approach or avoid an achievement-oriented activity can be summarized as:

$$T_A = (M_S - M_{AF}) \left[P_S \times (1 - P_S) \right]$$

Here $T_A = T_S + (-T_{AF})$ or the tendency to approach the task plus the tendency to avoid the task;

M_S equals motivation to succeed;
M_{AF} equals motivation to avoid failure; and
P_S equals the perceived probability of success.

Measures of these variables are usually converted to Z-scores to make intra- and inter-individual comparisons possible. The theoretical question is, of course, whether or not these determinants are a sufficient basis on

which to account for need-related behavior.

6. Weiner (1972) has proposed that by assigning weights to the determinants of behavior we can test the model against empirical data. The additional variables of Risk Preference, Level of Aspiration and Persistence of Behavior in Progress can also be tested.

7. M_S and M_{AF} are considered to be relatively stable personality traits, while incentive values of the goal are dependent on P_S. Thus future actions are almost entirely dependent on the effects of success and failure or on P_S which is the only variable that is free to fluctuate from trial to trial and which can therefore be classified as an experiential variable - that is, it depends, in part at least, on an individual's history of success.

8. Studies which have attempted to use the basic model outlined in 4 above have been unable to accommodate the effects of the history of success and failure associated with P_S. In order to make the model fit empirical data, Atkinson and Cartwright (1964) introduce an additional variable which they describe as "the inertial tendency" or the "unsatisfied tendency" to persist in achieving a goal (T_{Gi}). Thus a general tendency to achieve a goal may be added to the need to achieve and this additional energy affects the subsequent motivation to achieve success. Thus:

$$T_S = (M_S \times P_S \times T_S) + T_{Gi}$$

Given the complexity of the relationship among determinants of need-achievement behavior, it becomes obvious that simple models will not produce adequate solutions. It is nevertheless of interest to produce models which will account for behavior and which will match reasonably well the empirical findings reported in this area. If a model, using the determinants described

in the achievement model, can be structured so as to reproduce empirical data, then some measure of validity will accrue to the achievement model. Furthermore, the question of optimality will become more meaningful because the effects of changes in any of the determinants of need-related behavior can be pre-determined.

In order to test this assumption and the validity of our model, we selected data from a research study reported by Feather (1966). A brief summary of this study is presented here.

Feather notes that the effects of success and failure on subsequent performance are extremely complex, depending on the characteristics of the person and on the situation. In order to facilitate the disentanglement of determiners of action, he divided his subjects into four groups as follows:

1. High expectation - initial failure (H-F). Instructions induced a high initial expectation of success. These subjects were then given 5 items which were insoluble followed by 10 items at a .50 level of difficulty. The assumption was made that failure on the first five items would reduce P_S to .50.

2. Low expectation - initial failure (L-F). In order to induce a low expectation of success, instructions implied that the subjects would find the items difficult. Thus five insoluble items were followed by 10 items at the .50 level of difficulty. Prior failure should have the effect of lowering P_S.

3. High expectation - initial success (H-S). Subject instructions implied that the tests would be easy. Thus 5 easy items were followed by 10 items at a .50 level of difficulty. The assumption was that initial success should raise P_S even more.

4. Low expectation - initial success (L-S). The conditions were as in 3 above except that subjects were informed that the test items were difficult. Initial success should result in a rise of P_S to .50.

The conditions arranged by Feather were intended to elicit data about the effects of these four experimental conditions on subjects' performance and the relationship of this performance to differences in achievement and test anxiety. Thus initial success and failure should result in changes in the tendency to perform the task. According to Feather, this tendency consists of a <u>total motivation</u> made up of the tendency to achieve success, the tendency to avoid failure and extrinsic motivation.

> Among subjects in whom $M_S > M_{AF}$ the resultant tendency to perform the task is maximum when $P_S = .50$. Among subjects in whom $M_{AF} > M_S$ the resultant tendency to perform the task is minimum when $P_S = .50$. Thus, in contrast to Atkinson's earlier position (Atkinson, 1957), the present assumptions do not imply that a subject in whom $M_{AF} > M_S$ should "try hardest" when $P_S = .50$. On the contrary, because of the strong inhibitory tendency to avoid undertaking the task when $P_S = .50$, the resultant tendency to perform the task would be at a minimum for such a subject. (p. 288)

Using these basic assumptions, Feather states the following hypotheses:

1. Where $M_S > M_{AF}$ the resultant tendency to perform after the first 5 items will be stronger in the H-F and L-S conditions. This tendency should remain at a high level because the reamining 10 items were at .50 level of difficulty and therefore close to the values developed during initial trials.

2. Since it is assumed that higher resultant tendency determines superior performance, it can also be postulated that this group should obtain higher scores on the last 10 items.

3. Where $M_{AF} > M_S$ the resultant tendency to perform the task after the first 5 items will be greater for the L-F and H-S groups.

4. These two groups should also be relatively less successful.

By using a series of items it was possible to study changes in expectations over time. The questions of interest raised in this study are summarized by Feather as follows:

> Do estimates of expectation of success change more rapidly after failure than after success? Is the rate of change a function of the initial estimate of expectation of success, of n Achievement, and of Test Anxiety? Are there differences between subjects in the degree to which "typical" changes in probability estimates occur after success and failure? (p. 289)

The subjects in the study were 96 female undergraduates. Subjects completed protocols for 6 of the n Achievement pictures and the TAQ. The scores on these tests were converted to Z-scores and the difference between the two calculated for each S. In S's for whom this difference was positive it was assumed that $M_S > M_{AF}$; for those which were negative, it was assumed that $M_{AF} > M_S$.

The criterion test used in the study consisted of a series of 15 anagrams. For the H-F and L-F groups the first 5 anagrams were insoluble; for the H-S and L-S groups the first 5 were very easy. For all groups the remaining 10 items were rated as being at the .50 level of difficulty. Instructions varied as indicated above.

The data which we used as the basis for our analysis are presented below.

Table 1 presents the mean of the probability estimates obtained from subjects prior to attempting each of the anagrams. An analysis of variance of these data was also conducted by Feather. It demonstrated significant effects due to initial experience and an interaction between initial experience and trials.

Table 1

Adapted from Feather, 1966, p. 290

Mean Estimates of Probability of Success for Anagrams over 15 Trials

Group	Trials														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
H-F															
Success oriented (N = 9)	.61	.49	.41	.31	.23	.16	.27	.31	.28	.31	.32	.24	.27	.34	.35
Failure oriented (N = 9)	.70	.53	.38	.30	.22	.21	.29	.36	.51	.45	.52	.42	.43	.49	.49
L-F															
Success oriented (N = 9)	.56	.49	.38	.35	.29	.23	.33	.33	.35	.33	.31	.35	.33	.34	.35
Failure oriented (N = 9)	.55	.36	.28	.19	.15	.15	.18	.19	.26	.19	.23	.23	.24	.36	.31
H-S															
Success oriented (N = 9)	.67	.66	.69	.74	.75	.79	.68	.72	.69	.70	.68	.67	.63	.64	.63
Failure oriented (N = 9)	.54	.58	.66	.70	.71	.68	.66	.60	.53	.53	.56	.56	.56	.56	.55
L-S															
Success oriented (N = 9)	.50	.63	.66	.74	.74	.78	.70	.60	.59	.61	.57	.60	.59	.55	.50
Failure oriented (N = 9)	.57	.62	.70	.76	.78	.80	.69	.75	.74	.65	.70	.69	.68	.69	.64

Table 2 presents information about the mean number of anagrams correctly answered out of the final 10 anagrams. An analysis of variance of these results indicated that initial experience had a significant effect on performance.

Table 2

Adapted from Feather, 1966, p. 294

Number of Subjects Solving Anagrams on Trials 6-15

Group	Trials									
	6	7	8	9	10	11	12	13	14	15
H-F										
Success oriented (N = 9)	4	4	5	4	3	0	3	5	5	2
Failure oriented (N = 9)	5	5	5	5	7	3	4	3	5	3
L-F										
Success oriented (N = 9)	5	5	3	1	4	5	2	6	2	5
Failure oriented (N = 9)	2	6	7	1	9	5	6	7	4	4
H-S										
Success oriented (N = 9)	5	5	5	6	7	6	3	7	4	6
Failure oriented (N = 9)	5	3	5	5	7	6	6	5	4	6
L-S										
Success oriented (N = 9)	4	6	7	6	4	7	4	7	6	4
Failure oriented (N - 9)	5	6	6	3	7	8	7	6	4	6

Several other analyses were conducted by Feather. In this case, however, we are interested mainly in probability estimates and actual achievement and the way in which they interact as a system. The interested reader may wish to refer to the Feather article for more details and to compare these to our findings.

In his "Discussion" Feather points out that the major contribution of this report lies in the fact that a detailed analysis of probability estimates was made during the entire course of student responses. He also notes that the findings lend support to the belief that success and failure play a significant role in shaping estimates of probability of success and thus in expectation of success. He also notes that:

1. Probability estimates change more after failure at the first 5 items than after success at the first 5 items, suggesting that consistent failure has a greater effect than consistent success when initial expectation is intermediate.

2. Success-oriented subjects tend to make more "typical" changes in P_S estimates under conditions of success while failure-oriented subjects make more typical adjustments under failure conditions. (All of this may, of course, be related to past experience or inertial tendencies discussed above.)

3. Since this study was limited to eliciting P_S estimates from only one trial to the next, it is limited in terms of generalizability to situations in which trial-to-trial expectations are somewhat independent. S's were not asked to predict their chances of attaining a fixed performance level. On the other hand, the results obtained appear to conform to those found in other studies which looked at the effects of success and failure on expectations of success.

4. Initial experience has a significant effect on performance when S's are given feedback regarding success and failure. As in any experiment, it is difficult to generalize the results because the number of success and failure experience required for long-term increments and decrements in

achievement patterns for the various groups did not conform to expectation. For example, Table shows that the mean scores on performance in the initial failure condition are higher than those for failure-oriented subjects. It is not clear why this should be the case, and it may be that the models being used are not sufficiently developed to account for this anomaly.

5. Instructions given at the beginning of a series of tasks tend to set the general difficulty of the tasks at the outset so that P_S tend to cluster about a certain value depending on the instructions. Previous research also indicates that performance is related to this general level of difficulty because S's work harder when their expectation of success is intermediate. If the difficulty of the task is truthfully represented, task performance and initial probability tend to be correlated. In addition, S's tend to draw upon past experiences and to make judgments accordingly (This has the effect of anchoring them to reality so that fluctuations about the mean in terms of estimates of probability of success, level of aspiration and responses to success and failure are kept within reasonable bounds.)

The Modeling Procedure

Data from the Feather study were examined to determine how achievement (A) and probability of success (P) varied over time for each treatment group. Both A and P were plotted against the number of trials in each instance. As well, the number of successive trials during which P or A continued to change in the same direction and the number of trials before the trend reversed was examined and the results pooled for all eight groups. P was found to change an average of .07 in a positive direction and an average of .07 in a negative direction in 1.74 trials. A was found to change an average of 38 per cent in a positive direction and 35 per cent in a negative direction in 1.70 trials. Further, the change in P was opposite in direction to the change in A at any time. For purposes of the model, P was taken to change 0.10 in two trials and A 40 per cent in two trials. The time taken in the Feather study was estimated at two minutes per trial because 15 anagrams were to be completed in 30 minutes.

The following interactions and their relationships were incorporated into the model of this system:

1. Probability of success and incentive were represented as hypothesized by Feather, or as $P \times I.(P \times (1-P))$.

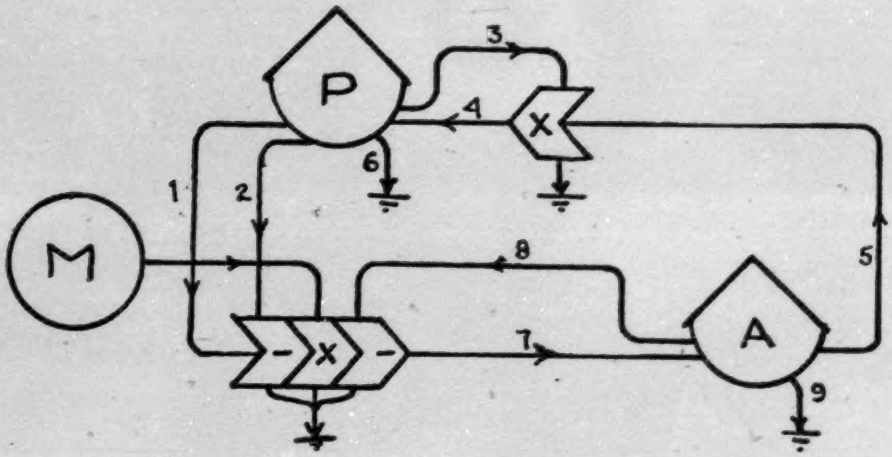
2. Incentive and probability of success interact with motivation as hypothesized, or $M \times P \times (1-P)$.

3. The additional effect of the persistence of previous achievement activity or the "inertial tendency" adds an inverse variation component to the interaction of $M \times P \times (1-P)$. This relationship derives from Feather's data which was mentioned previously.

4. Probability of success at time t + 1 is directly proportional to the magnitude of achievement and probability of success at time t.

The system and the interactions which occur within the system are shown schematically in Figure 2. The symbols are from Odum (1971).

Figure 2



The dynamic variables are shown with "tank-like" structures; the interactions by "arrows"; and the contribution to and products of interactions shown by the lines. Interactions in which variation is direct are shown with a' x ' and inverse variations with a' - '. Motivation (M) is considered outside the system, that is, it is considered a relatively stable personality characteristic and for purposes of the model does not vary over time. The numbers on each line indicate the number of the constant associated with the contribution or product and will also be used to identify these.

In translating the symbolic representation of this system to a useable mathematical form, the following conventions are followed:

1. The magnitude of a variable can assume values between zero and one. In other words, the magnitude of the variable is always represented

as a proportion of the maximum possible magnitude.

2. By using convertion #1, inverse variation can be represented mathematically as one minus the magnitude of the variable which varies inversely.

3. The contribution of any variable to an interaction, or the effect of the result of an interaction is dependent on the magnitude of all interacting variables. Thus an interaction of A and P in which the relationship is a direct variation would be A x P. The actual magnitude of the contribution (or product) is modified by a constant (termed an interaction coefficient). Therefore, several interaction contributions may have the term A x P but these differ when the interaction coefficient is considered. The coefficient has two functions - one, it indicates the magnitude of the total possible contribution or product, and second, its reciprocal is a time constant. The time constant indicates the relative time period for one complete cycle.

4. Constants 6 and 9 are associated with the depreciation of the variable over time. This depreciation depends only on the magnitude of the variable. The reciprocal of the constant indicates how long it would take for the variable to depreciate to zero if there is no maintenance of the variable.

Table 5 shows each of the contributions or products and how each is represented mathematically.

Table 5

Mathematical Representation of

Contributional Products

Constant #	Representation	Description
1	$k_1 M(P)(1-P)(1-A)$	contribution of probability of success
2	$k_2 M(P)(1-P)(1-A)$	contribution of incentive
3	$k_3 PA$	contribution of P to change in P
4	$k_4 PA$	product of interaction of P and A
5	$k_5 PA$	contribution of A to change in P
6	$k_6 P$	depreciation of P
7	$k_7 M(P)(1-P)(1-A)$	product of interaction of M, P, incentive & A (inertial tendency)
8	$k_8 M(P)(1-P)(1-A)$	contribution of A to interaction of M
9	$k_9 A$	depreciation of A

The mathematical representation of the system is accomplished by developing a differential equation for each variable. This equation is a sum of all the products which contribute to the variable minus the sum of all contributions the variable makes to interactions and the depreciation of the variable. The two differential equations representing this system are:

$$P = k_4PA - k_3PA - k_1 M(P)(1-P)(1-A) - k_2 M(P)(1-P)(1-A) - k_6P$$

$$A = k_7 M(P)(1-P)(1-A) - k_8 M(P)(1-P)(1-A) - k_5PA - k_9A$$

If these equations are integrated simultaneously from zero to time t, the result is a plot of the changes of the variables during time t. Our procedure has been to use an analog computer to do this. However, digital

computers can be used if the equations are rewritten as:

$$P_{t+1} = P_t + i\,[k_4 P_t A_t - K_3 P_t A_t - k_1 M_t (P_t)(1-P_t)(1-A_t) - k_2 M_t (P_t)(1-P_t)(1-A_t) - k_6 P_t]$$

$$A_{t+1} = A_t + i[k_7 M_t (P_t)(1-P_t)(1-A_t) - k_8 M_t (P_t)(1-P_t)(1-A_t) - k_5 P_t A_t - k_9 A_t]$$

where i is some time interval. It must be noted that i must be kept small in order to achieve accuracy in most cases.

Because the constants are time dependent and to date, no feasible technique has been developed for relating raw data to the constants, an alternative is to search for the constants and to refine these by successive approximation. This is possible because the behavior of the variables is understood for some time period at least.

A computer program (using the equation for digital machines) can be written to solve for P and A by successive iterations and given the initial values for P, A, and M. The time increment should be kept small (approximately .05 in this case) as accuracy increase with small values of i. In this case, approximately 40 iterations would be necessary to cover the two minute time span. The program can be adopted to test all values of all combinations of constants in steps of .1 for each constant. In order to reduce the amount of computer time A and P should be checked after each iteration. If either value is greater than 1 or less than zero, the constants used for this iteration are incorrect and no further iterations are useful. When a solution is attained, it should be subjected to more iterations to assure that it remains viable over time, otherwise further refinement is necessary.

We have found the solutions obtained have a unique set of constants. Some adjustment of the constants are necessary in order to obtain the proper range of oscillation of the variables or to assure that the variable level at the proper magnitude. In this respect, an analog computer is easier and faster to work with but still not a requirement. If no feasible solution is reached when all combinations of constants have been tried, i should be decreased. In this case, the variables may be fluctuating very rapidly and an increment in time (i) which is too large does not allow the equations to respond adequately.

It should also be noted that multiples of the constants also provide a solution, that is, if all constants are multiplied by 2, for example. Determination of the constant which apply to the model can best be ascertained by taking the reciprocal of the constants to obtain the time constant. These can be checked to see if they are reasonable. For example, the time for depreciation of variables should be reasonable. If these are too large or too small by some factor, all constants should be adjusted by that factor.

Discussion

Theoretical Implications

In order to understand some of the information derived from the model, a more detailed discussion of the interaction coefficient and the time constant may be beneficial. These two values are dependent on the nature of the variables which interact or are affected by the interaction and also on the nature of the contributions or products themselves.

The time constant is dependent on the capacitance of the variable concerned and the resistance encountered in contributing to an interaction or in affecting a variable (or $T = RC$). Capacitance in a social science context is a measure of the amount of input which can be accommodated over a period of time and is inversely proportional to the difficulty in achieving integration of the input. The resistance is the difficulty encountered when a variable contributes to an interaction or the difficulty encountered in utilizing the product of an interaction. In either case, the difficulty is determined by the nature of the interaction taking place. The time constant, then, is an indication of the time required to produce some specified amount of change, given the nature of the variable's ability to accommodate interaction within the system and the difficulty in achieving interactions. Thus if the specified amount of change is 20 per cent all of the time constants, when compared to each other, indicate the ratios of times required to produce such a change. The interaction coefficient is the reciprocal of the time constant, and while this value is required for the equations in the model, it is the time constant which provides the information of interest.

Table 6 shows the interaction coefficients which correspond to the interactions and products indicated previously in Table 5 and in the schematic diagram of the system. Also shown are the time constants and the ratio of each time constant to the smallest time constant (time constant #7).

Table 6

Constants for the System

Number	Interaction Coefficient (k)	Time (T) Constant	T_i/T_7 (Approximate)
1	6.80	0.15	3
2	6.80	0.15	3
3	2.72	0.37	7
4	4.44	0.23	5
5	1.15	0.87	16
6	0.70	1.43	29
7	20.00	0.05	1
8	1.10	0.91	18
9	0.01	100.00	2000

The coefficients duplicate (with some qualifications) the results obtained by Feather (1966) for those conditions which applied to the individuals used for that study. Plots of the changes in P and A over the series of trials conducted by Feather do not result in a series of smooth curves as is the result from the model. This is to be expected since the sample in the Feather study was quite small. However, the nature of the changes are duplicated for the different conditions as are the magnitudes of the fluctuations. It can be seen that the fluctuations decrease as the number of trials increase (as was the case in the Feather study) and that the ultimate leveling of P and A occurs at a value corresponding to the mean for that sample. With a large sample, curve fitting techniques could be applied to the raw data to achieve plots which are regular and thus determine precise reliability of the

results obtained from the model.

A number of the plots shown in Appendix A apply to situations which did not exist in the model. These are values for P, A, and M which did not exist in the sample but for which predictions could be made from achievement theory. In these cases, the results from the model do correspond with predictions.

Examination of the ratios of various time constants provides some useful information. When the contribution of P to future changes in P (constant 3) are compared to the contribution of A (constant #5) to this change, the ratio of T_5 and T_3 is about 2:1. This indicates that changes in P are more readily affected by the magnitude of P than the magnitude of A. In other words, changes in P are facilitated more easily by manipulating P rather than A. Given the nature of P and A and the nature of the interaction, the utilization of changes in P to produce changes in P over time proceeds more readily than attempting to change P over time by changing A.

The magnitude of T_8, the inertial effect of achievement, when compared to the magnitude of T, (probability) and T_2 (incentive) in achieving the interaction necessary for increases in achievement indicate the longevity of the inertial effect. The depreciation of P, or the rapidity with which it decreases if not maintained, indicates that P is most task-specific and changes very rapidly. Using graph #14, which is a plot of the behavior of P when maintained and not maintained, the indication is that P would be reduced to half its value in about two minutes. The reverse is true of A. Here the indication is that the retention of achievement level is still 50 per cent after approximately 30 minutes.

Psychological Implications

The advantage of using a model such as we have described have already been noted. The question arises, however, as to whether or not such a model can represent empirical data faithfully, whether it fits existing theory, and whether it can give us new psychological information or at least suggest research hypotheses which may lead to clarification of psychological principles. It is the feeling of the authors that all three of these aims can be encompassed.

First of all, an examination of the graphs presented in Appendix A indicate substantial agreement between our results and those obtained by Feather. (Note that we did not use his data in deriving our model but used it only as an empirical validity check). Thus, it can be seen that probability estimates change more after failure at the first five items than after success. Success and failure also function to bring expectation and achievement into line. This is true, of course, only as long as motivation remains at a reasonable level. Initial experience has a strong effect on performance when subjects are given feedback in terms of success and failure. Finally, there is also evidence that task performance and initial probability are correlated. We do note, however, that our model suggests that achievement is at its highest level when expectation of success and achievement are both at a relatively high level. This finding appears to be contrary to Feather's and needs further investigation.

Secondly, we note that the graphs demonstrate the validity of achievement theory as it is currently understood. Thus expectation of success interacts with achievement such that a balance is achieved between the two as long as motivation remains high. Furthermore, it can be seen that

"inertial tendencies" operate to keep achievement up even when expectation of success is very low. We also note that feedback effects operate quite rapidly so that expectation of success is affected almost immediately after results have become available and estimates of probability of success adjust rapidly when feedback is given.

Finally, we address ourselves to the possibility of implications for further research. Numerous possibilities present themselves, but some of the most salient can be noted here.

1. Psychologists have long referred to optimal conditions for the support of specific activities. For example, it is well established that anxiety and performance on tests have some optimal relationship. Unfortunately, establishing optimal levels of anxiety creates a problematic situation because no acceptable measure is known. In the model we have presented it is relatively simple to determine the effect of increasing any of the sources of input and thus pre-determining the effects of increasing power in any one variable.

2. Expectation of success and error rate have frequently been the bases for study in relation to programmed instructional formats. There is some suggestion in our model that when achievement and expectation of success are too high solutions become infeasible. Further study is required to elucidate this problem. The Feather data could not be used in this case because the subjects used formed a group which was too homogeneous. We intend to collect further data with more heterogeneous subjects to study this possibility.

3. A number of other psychological relationships can be investigated by the use of this model. The creativity-intelligence dichotomy, personality correlates and learning, and even the heredity-environment issue may possibly be addressed in this way.

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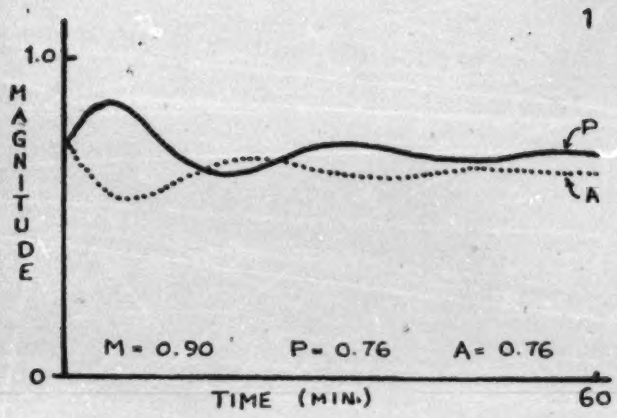
APPENDIX A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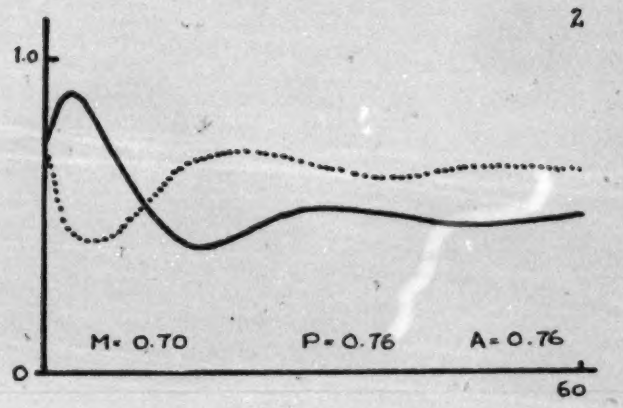
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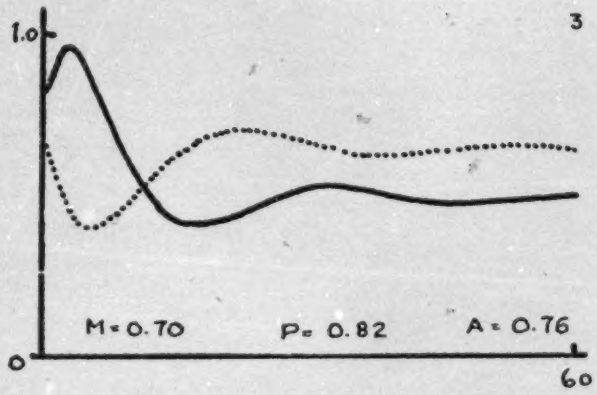
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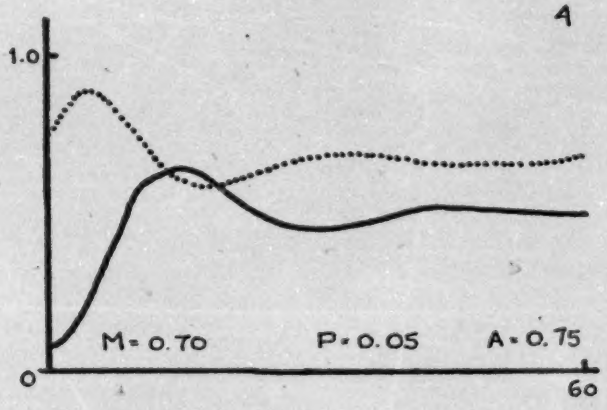
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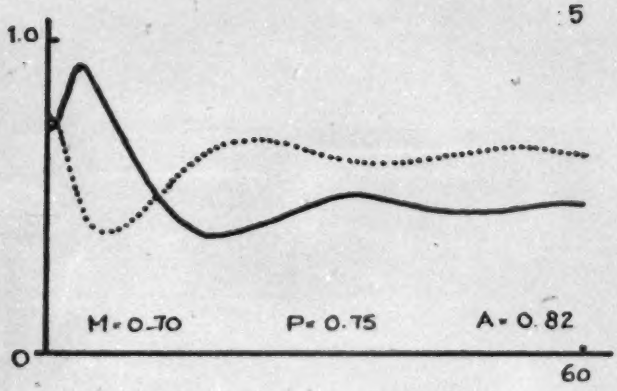
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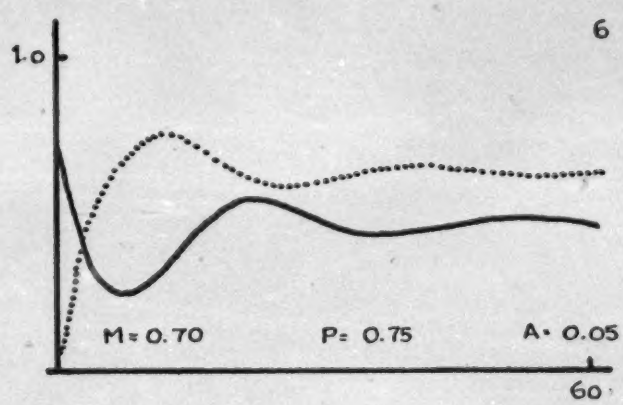
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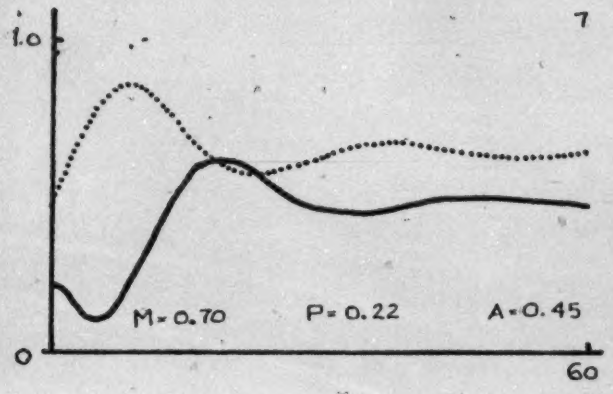
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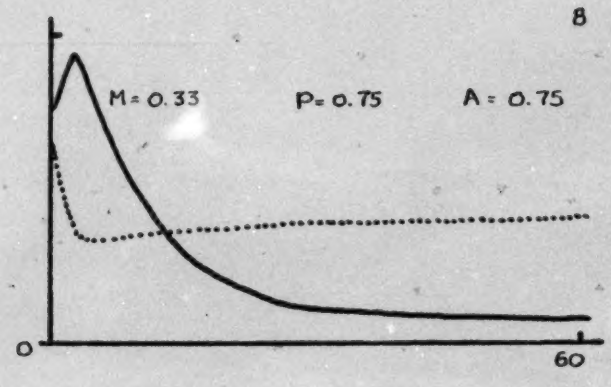
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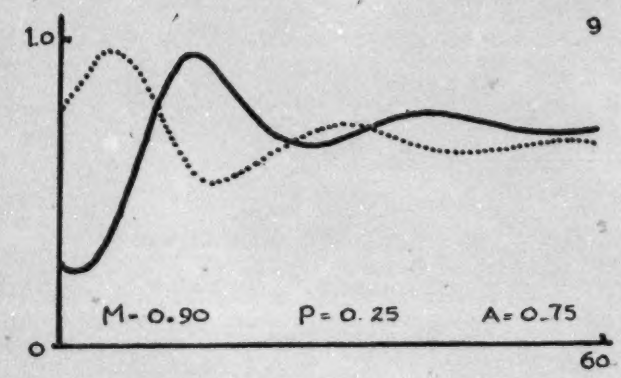
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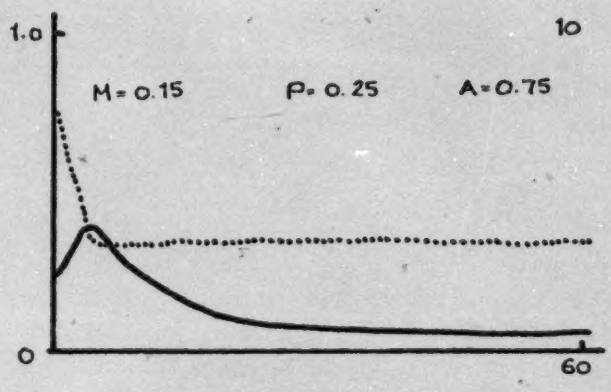
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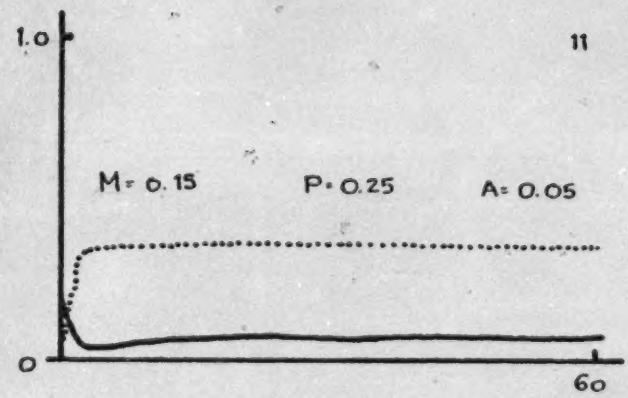
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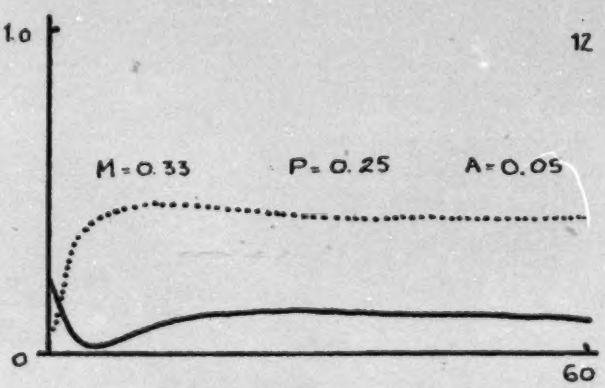
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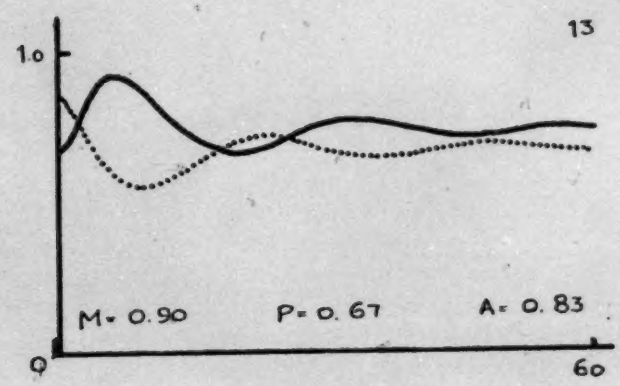
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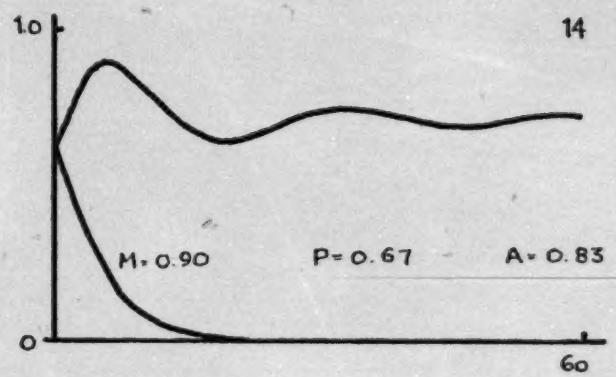
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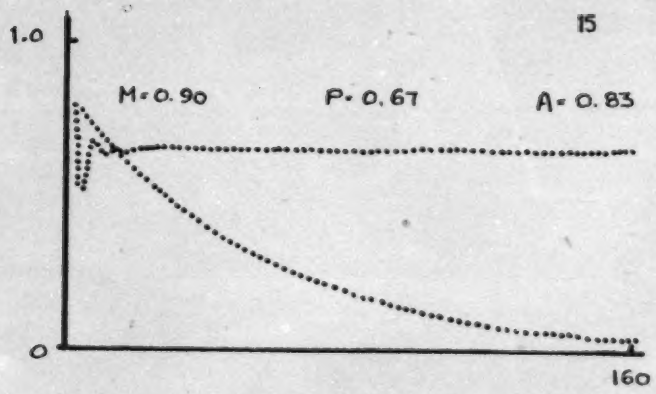
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